

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 1	Daily Tutorial Sheet - 3
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- 31.** $1 - \cos^2 A + \sin^2 B + \sin^2 C = 1 - (\cos^2 A - \sin^2 B) + \sin^2 C$
 $= 1 - \cos(A+B) \cos(A-B) + \sin^2 C = 1 + \cos C \cos(A-B) + 1 - \cos^2 C$
 $= 2 + \cos C (\cos(A-B) - \cos C) = 2 + \cos C (\cos(A-B) + \cos(A+B))$
 $= 2 + 2 \cos A \cos B \cos C = 2(1 + \cos A \cos B \cos C)$ Hence Proved
- 32.** $\cos(A+B+C) = \cos A \cdot \cos B \cdot \cos C - \sin A \cdot \sin B \cdot \cos C - \sin B \cdot \sin C \cdot \cos A - \sin C \cdot \sin A \cdot \cos B$
 Also, $A+B+C = \pi$ and divide the equation by $\sin A \cdot \sin B \cdot \sin C$
 $\Rightarrow \frac{\cos \pi}{\sin A \cdot \sin B \cdot \sin C} = \cot A \cot B \cdot \cot C - \cot C - \cot A - \cot B$
 $\Rightarrow \cot A + \cot B + \cot C - \operatorname{cosec} A \cdot \operatorname{cosec} B \cdot \operatorname{cosec} C = \cot A \cdot \cot B \cdot \cot C$
- 33.(B)** $(2 \cos^2 x)^2 - 2 \cos 2x - \frac{1}{2} \cos 4x = (1 + \cos 2x)^2 - 2 \cos 2x - \frac{1}{2} (2 \cos^2 2x - 1)$
 $= 1 + \cos^2 2x + 2 \cos 2x - 2 \cos 2x - \cos^2 2x + \frac{1}{2} = \frac{3}{2}$
- 34.** $\cos 2\theta = (\sqrt{2} + 1) \left(\cos \theta - \frac{1}{\sqrt{2}} \right) \Rightarrow 2 \cos^2 \theta - 1 = (\sqrt{2} + 1) \cos \theta - 1 - \frac{1}{\sqrt{2}}$
 $\Rightarrow (2 \cos \theta - 1) \cos \theta = \frac{1}{\sqrt{2}} (2 \cos \theta - 1) \Rightarrow 2 \cos \theta - 1 = 0 \text{ or } \cos \theta = \frac{1}{\sqrt{2}}$
 $\Rightarrow \cos \theta = \frac{1}{2} = \cos \frac{\pi}{3} \text{ or } \cos \theta = \cos \frac{\pi}{4} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3} \text{ or } \theta = 2k\pi \pm \frac{\pi}{4}$
- 35.** $\frac{\sin p\theta}{\cos p\theta} = \frac{\cos q\theta}{\sin q\theta} \Rightarrow \sin p\theta \cdot \sin q\theta = \cos p\theta \cdot \cos q\theta \Rightarrow \cos(p+q)\theta = 0$
 $\Rightarrow \cos(p+q)\theta = \cos \frac{\pi}{2} \Rightarrow (p+q)\theta = 2n\pi \pm \frac{\pi}{2} \Rightarrow \theta = \frac{1}{p+q} \left[2n\pi \pm \frac{\pi}{2} \right]$
- 36.** $\cos \left(\theta - \frac{\pi}{4} \right) = \cos \left(2\theta - \frac{\pi}{4} \right) \Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm \left(2\theta - \frac{\pi}{4} \right) \Rightarrow \theta - \frac{\pi}{4} = 2n\pi + 2\theta - \frac{\pi}{4} \text{ or } \theta - \frac{\pi}{4} = 2m\pi - 2\theta + \frac{\pi}{4}$
 $\Rightarrow \theta = -2n\pi \text{ or } 3\theta = 2m\pi + \pi/2 \Rightarrow \theta = -2n\pi \text{ or } \theta = \frac{2m\pi}{3} + \frac{\pi}{6}$
- 37.** $2 \sin 3\theta \cdot \cos 2\theta - \sin 3\theta = 0 \Rightarrow \sin 3\theta \cdot [2 \cos 2\theta - 1] = 0 \Rightarrow \sin 3\theta = 0 \text{ or } \cos 2\theta = \frac{1}{2}$
 $\Rightarrow 3\theta = n\pi \text{ or } 2\theta = 2m\pi \pm \frac{\pi}{3} \Rightarrow \theta = \frac{n\pi}{3} \text{ or } \theta = m\pi \pm \frac{\pi}{6}$
 Put $n = 0, 1, 2, 3$ and $m = 0, 1$ to get $\theta = 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$
- 38.** $(\sin x + \cos x)(1 - \cos x \cdot \sin x) - (1 - \sin x \cdot \cos x) = 0$
 $\Rightarrow 1 - \cos x \cdot \sin x = 0 \text{ or } \sin x + \cos x - 1 = 0$
 $\Rightarrow \sin 2x = 2 \text{ or } \sin \left(x + \frac{\pi}{4} \right) = \sin \frac{\pi}{4}$
 No solution or $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

39. $(4\cos^6 x - 3\cos^4 x) + (3\sin^4 x - 4\sin^6 x) = 0$

$$\Rightarrow 3(\sin^4 x - \cos^4 x) = 4(\sin^6 x - \cos^6 x)$$

$$\Rightarrow 3(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 4(\sin^2 x - \cos^2 x)(\cos^4 x + \sin^4 x + \sin^2 x \cos^2 x)$$

$$\Rightarrow \sin^2 x - \cos^2 x = 0 \quad \text{or} \quad 1 - 4\sin^2 x \cos^2 x = 0$$

$$\cos 2x = 0 \quad \text{or} \quad \sin^2 2x = 1 = \sin^2 \frac{\pi}{2}$$

$$2x = 2n\pi \pm \frac{\pi}{2} \quad \text{or} \quad 2x = m\pi \pm \frac{\pi}{2}$$

$$x = n\pi \pm \frac{\pi}{4} \quad x = m\pi \pm \frac{\pi}{4}$$

40. $2\sin x \cdot \cos x + \sin x + 2\cos^2 x + \cos x = 0$

$$\Rightarrow (2\cos x + 1)(\sin x + \cos x) = 0$$

$$\Rightarrow \cos x = -\frac{1}{2} \quad \text{or} \quad \sin\left(x + \frac{\pi}{4}\right) = 0 \quad \Rightarrow \quad x = 2n\pi \pm \frac{2\pi}{3} \quad \text{or} \quad x = m\pi - \frac{\pi}{4}$$

41.(B) $\sqrt{3} \cos x + \sin x = \sqrt{2} \Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$

$$\Rightarrow x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4} \quad \Rightarrow \quad \text{The most general solution is } 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$$

42.(C) $\cos^3 \theta = \frac{1}{4} [\cos 3\theta + 3 \cos \theta] \quad \dots (i)$

$$\cos^3 (120 + \theta) = \frac{1}{4} [\cos 3\theta + 3 \cos (120 + \theta)] \quad \dots (ii)$$

$$\cos^3 (240 + \theta) = \frac{1}{4} [\cos 3\theta + 3 \cos (240 + \theta)] \quad \dots (iii)$$

Adding (i) and (ii) and (iii), to get:

$$\begin{aligned} \cos^3 \theta + \cos^3 (120 + \theta) + \cos^3 (240 + \theta) &= \frac{3}{4} \cos 3\theta + \frac{3}{4} [\cos \theta + \cos (120 + \theta) + \cos (240 + \theta)] \\ &= \frac{3}{4} \cos 3\theta + \frac{3}{4} [\cos \theta + 2 \cos (180 + \theta) \cos 60^\circ] = \frac{3}{4} \cos 3\theta + \frac{3}{4} [\cos \theta + 2 \cdot \frac{1}{2} (-\cos \theta)] \\ &= \frac{3}{4} \cos 3\theta + 0 = \frac{3}{4} \cos 3\theta. \end{aligned}$$

43. (B) Consider $3 \tan \theta \cdot \tan \phi = 1, \quad \Rightarrow \quad \frac{1}{\tan \theta \cdot \tan \phi} = \frac{3}{1} =$

Apply 'C' and 'D' to get

$$\Rightarrow \frac{1 - \tan \theta \cdot \tan \phi}{1 + \tan \theta \cdot \tan \phi} = \frac{3 - 1}{3 + 1} \Rightarrow \frac{\cos (\theta + \phi)}{\cos (\theta - \phi)} = \frac{2}{4} \Rightarrow \frac{\cos (\theta - \phi)}{\cos (\theta + \phi)} = 2$$

44.(C) $\tan \theta = -1 \quad \Rightarrow \quad \theta = \frac{3\pi}{4}, \frac{7\pi}{4} \text{ in } [0, 2\pi]$

$$\cos \theta = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \theta = \frac{\pi}{4}, \frac{7\pi}{4} \text{ in } [0, 2\pi]$$

$\therefore$ Common value is $x = \frac{7\pi}{4}$ $\therefore$ General solution is $2n\pi + \frac{7\pi}{4}, n \in I$

45.(D) $4 \sin \theta \cos \theta - 2 \cos \theta - 2\sqrt{3} \sin \theta + \sqrt{3} = 0 \Rightarrow 2 \cos \theta (2 \sin \theta - 1) - \sqrt{3} (2 \sin \theta - 1) = 0$

$\Rightarrow (2 \sin \theta - 1)(2 \cos \theta - \sqrt{3}) = 0 \Rightarrow \sin \theta = \frac{1}{2}, \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$